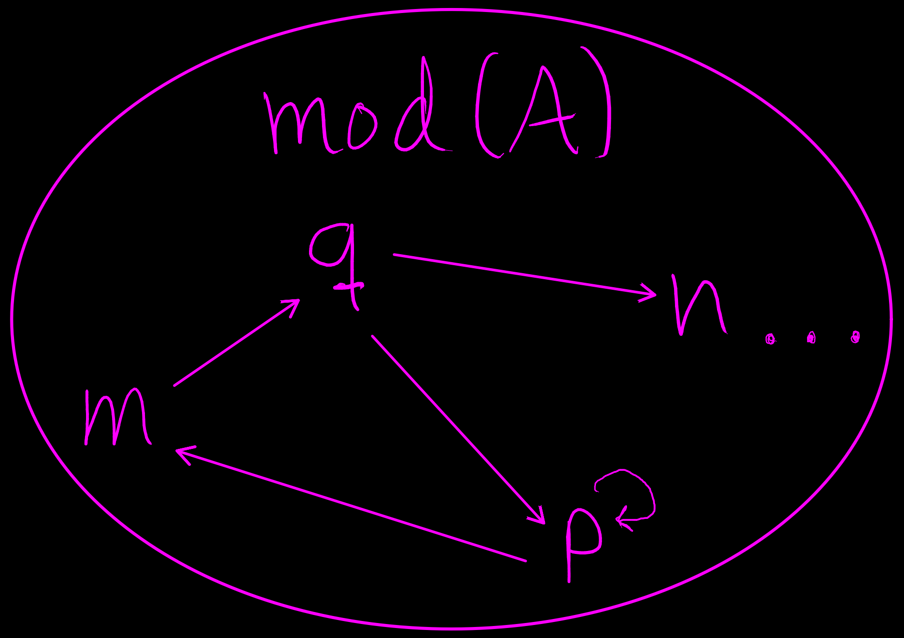
# Mutation of torsion pairs of small modules and silting subcategories

Diego Alberto Barceló Nieves

Škola v přírodě 2025

Kořenov, 14/11/25

# Category of A-modules



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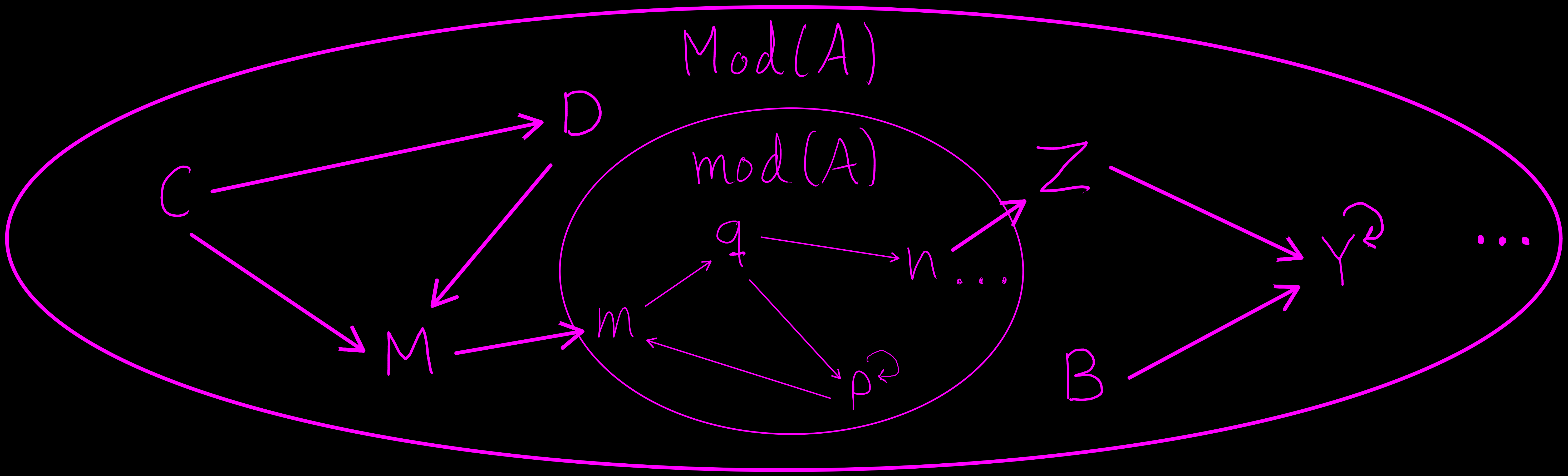
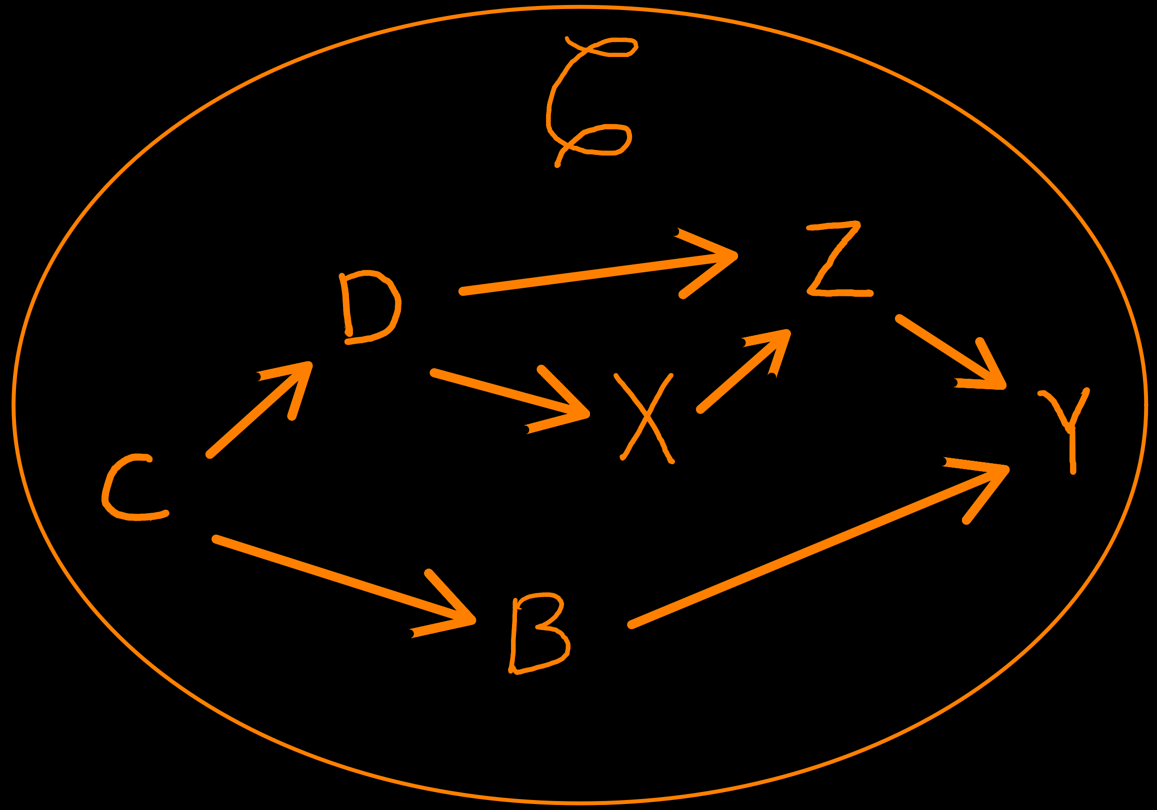


# Torsion pairs



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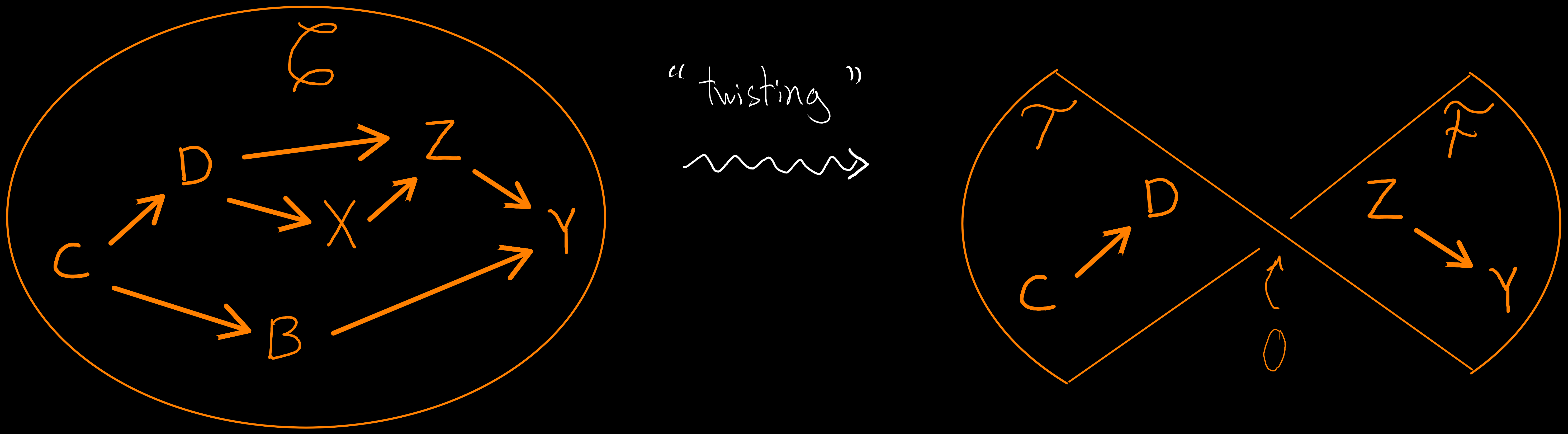

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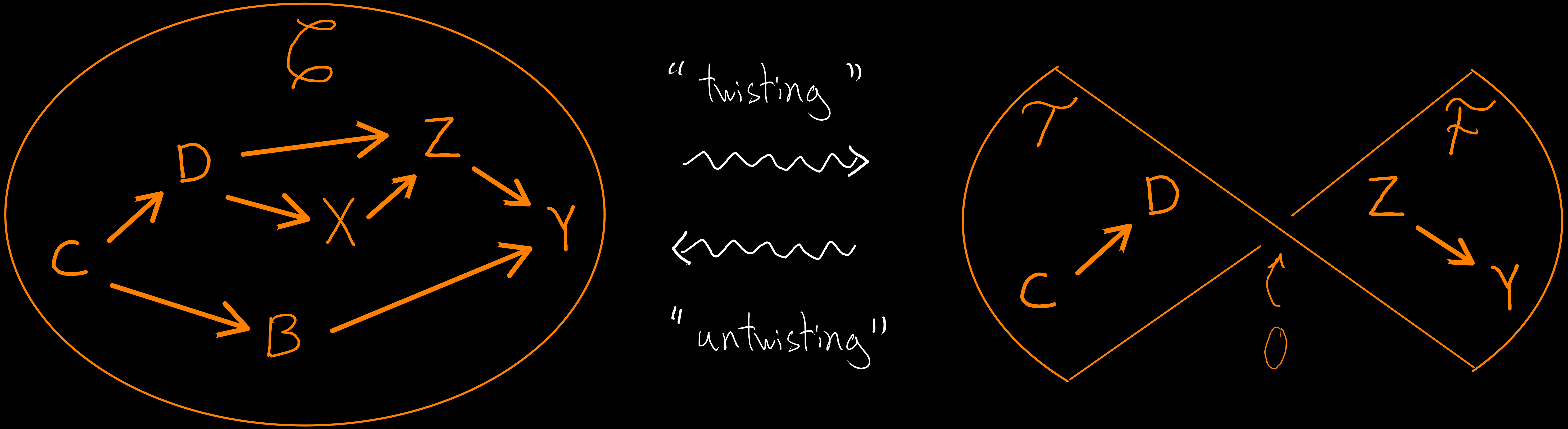
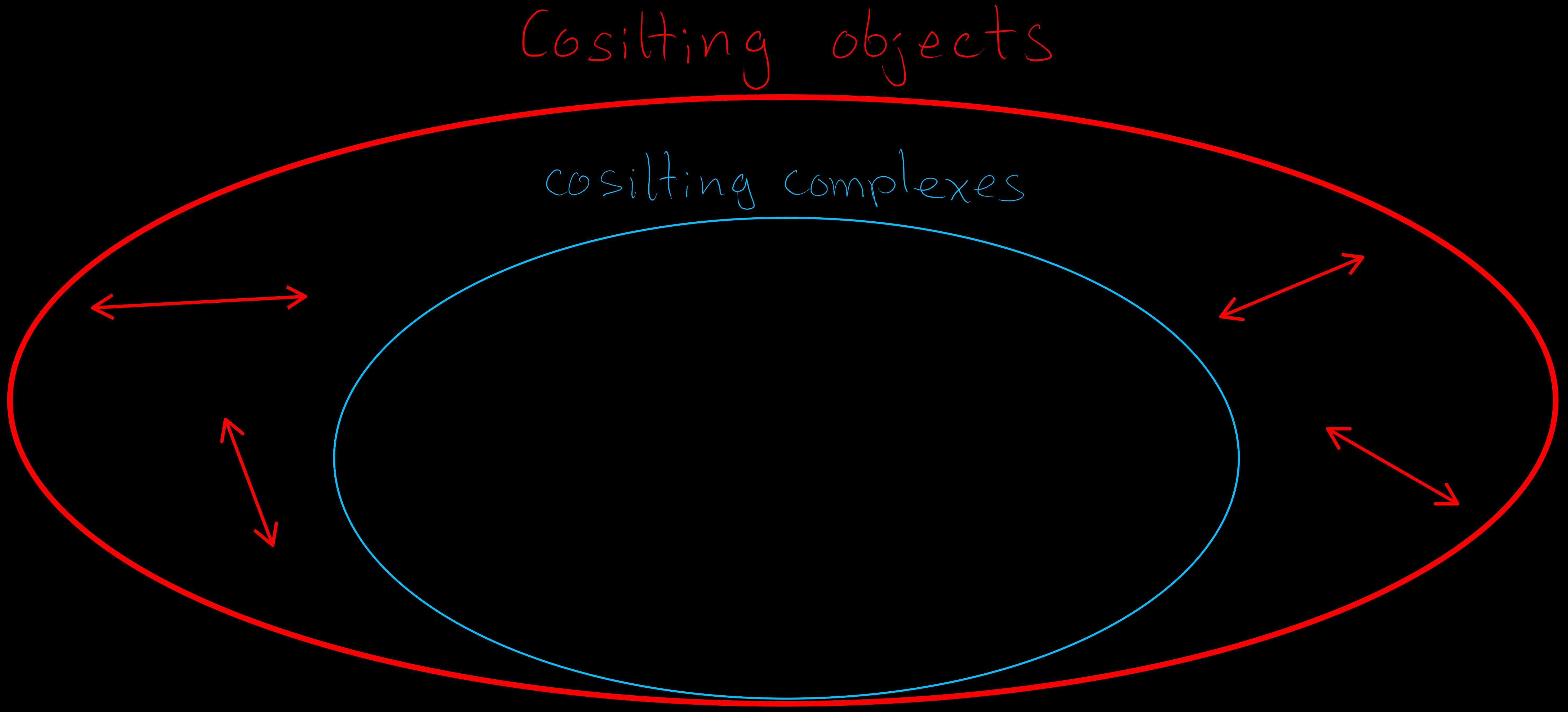


# Cosilting objects

Cositting objects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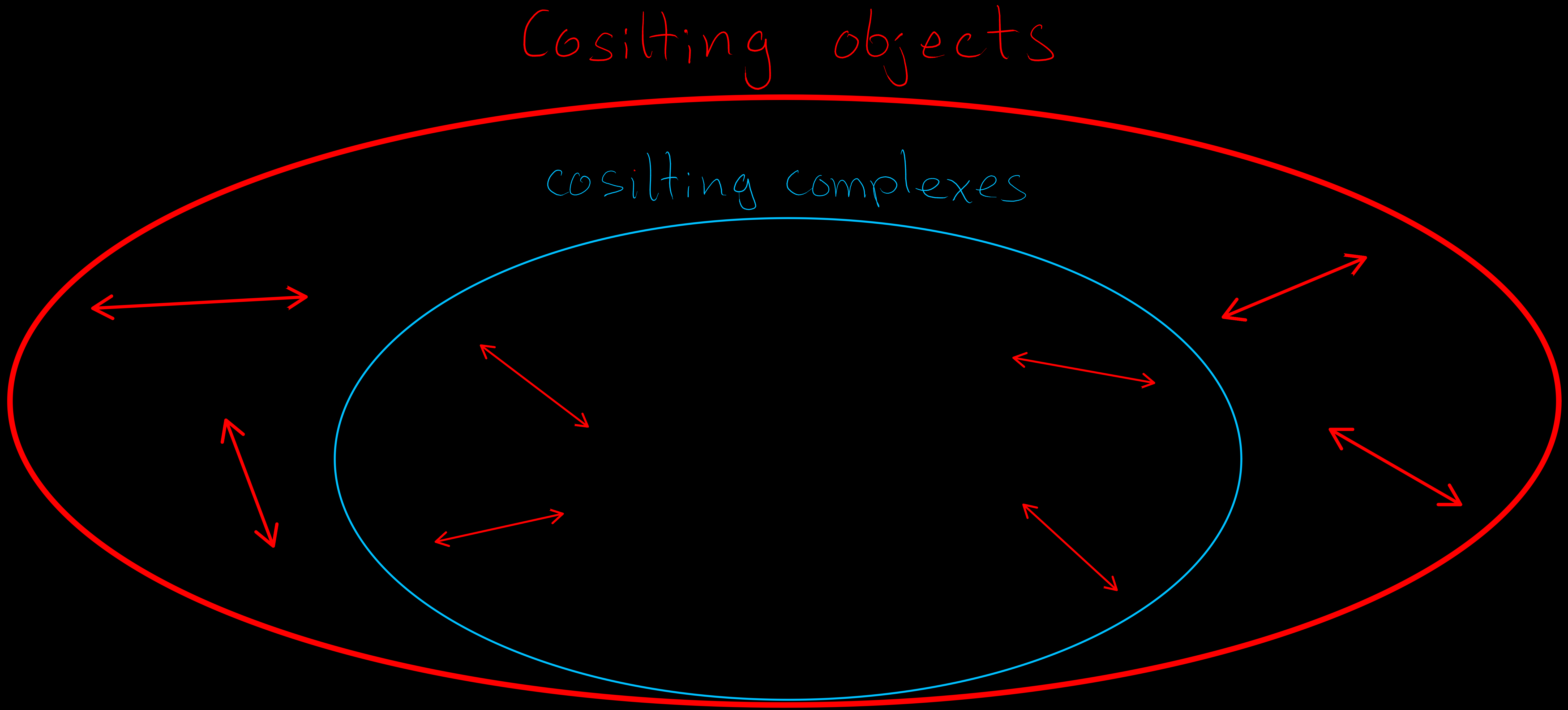
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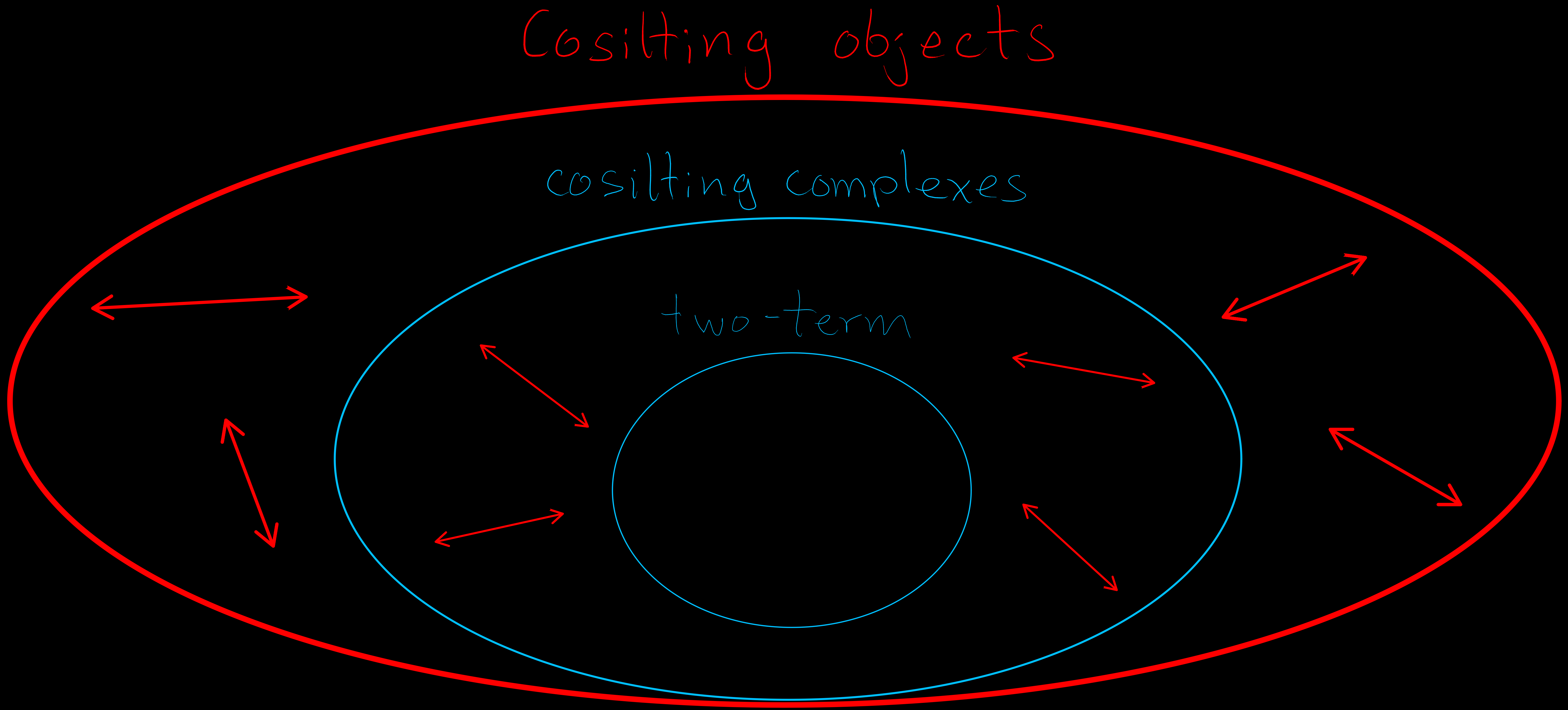
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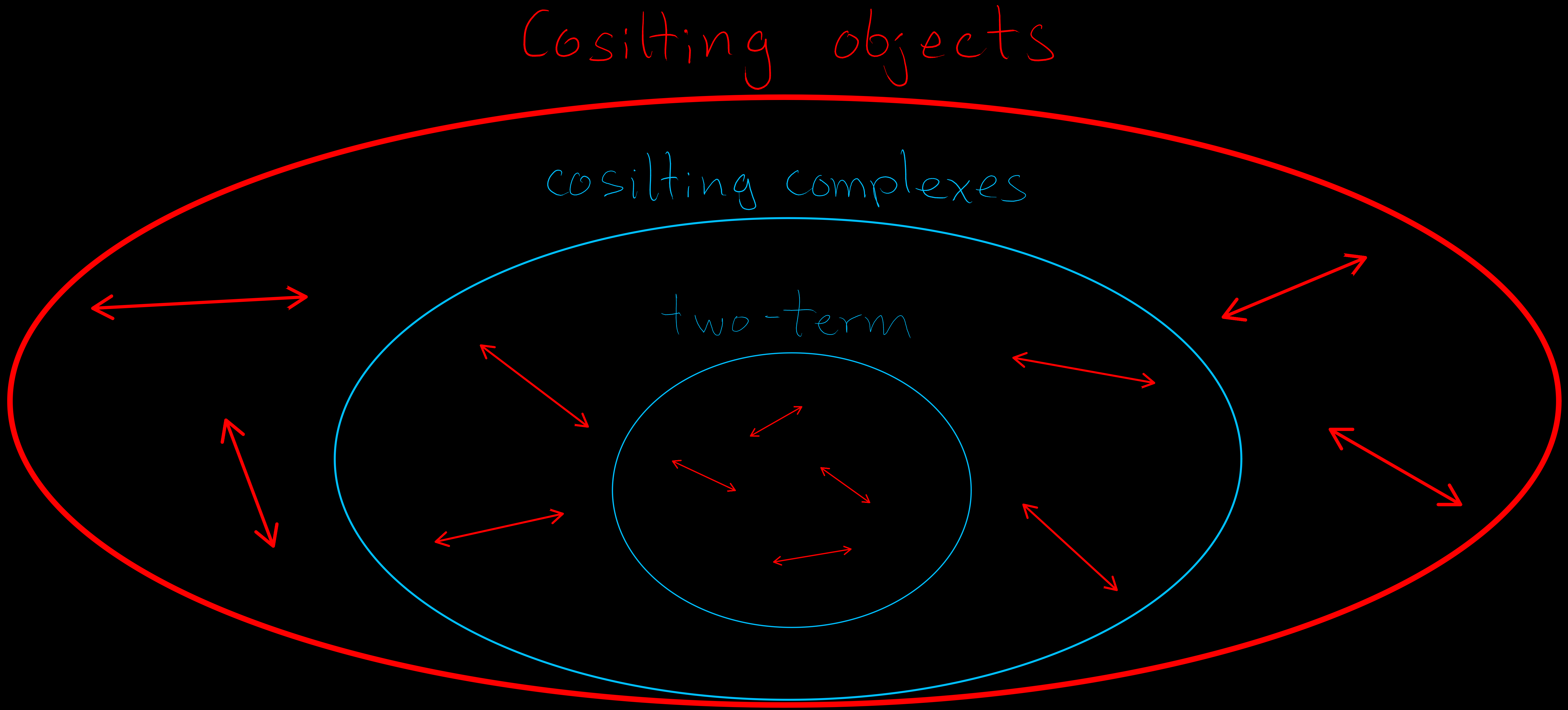
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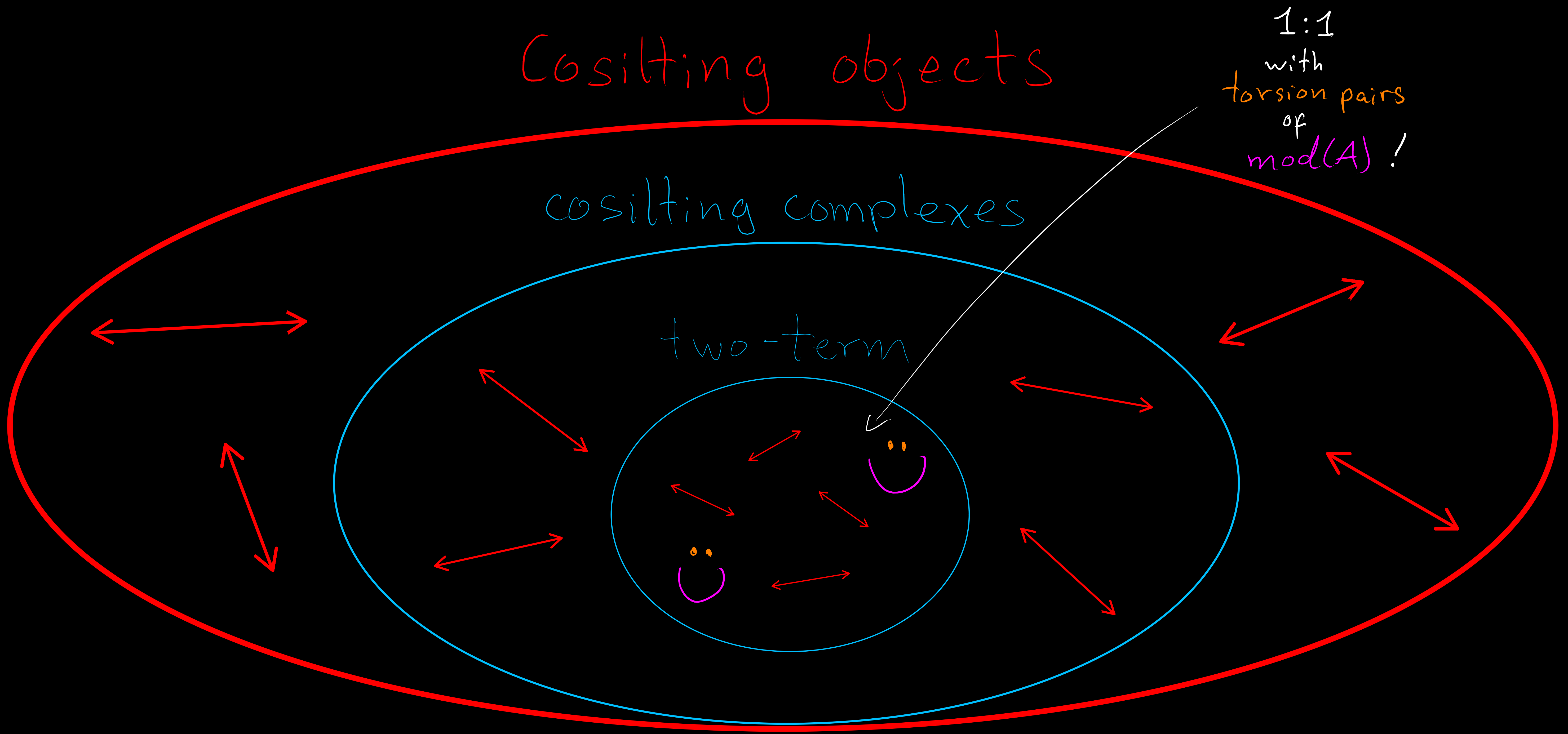
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1:1
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of
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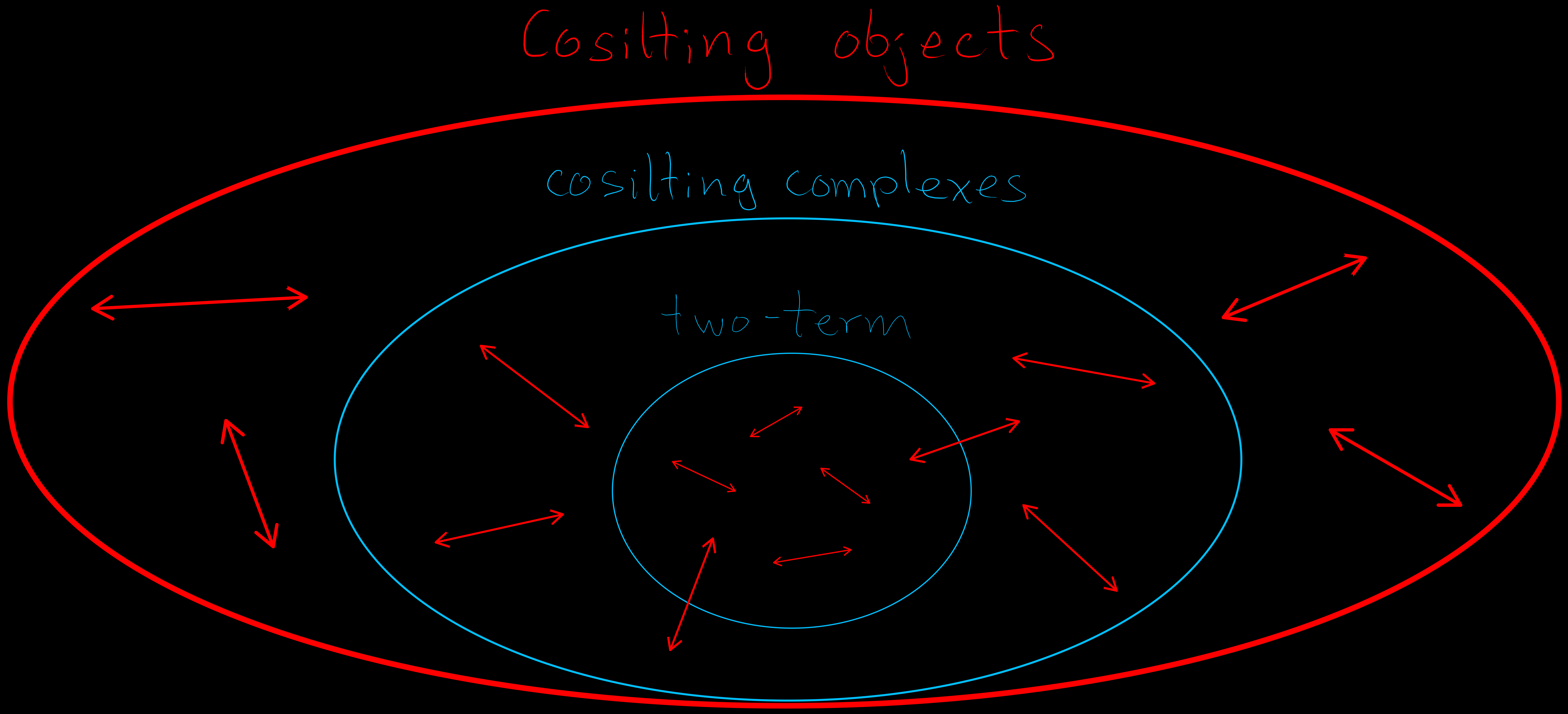
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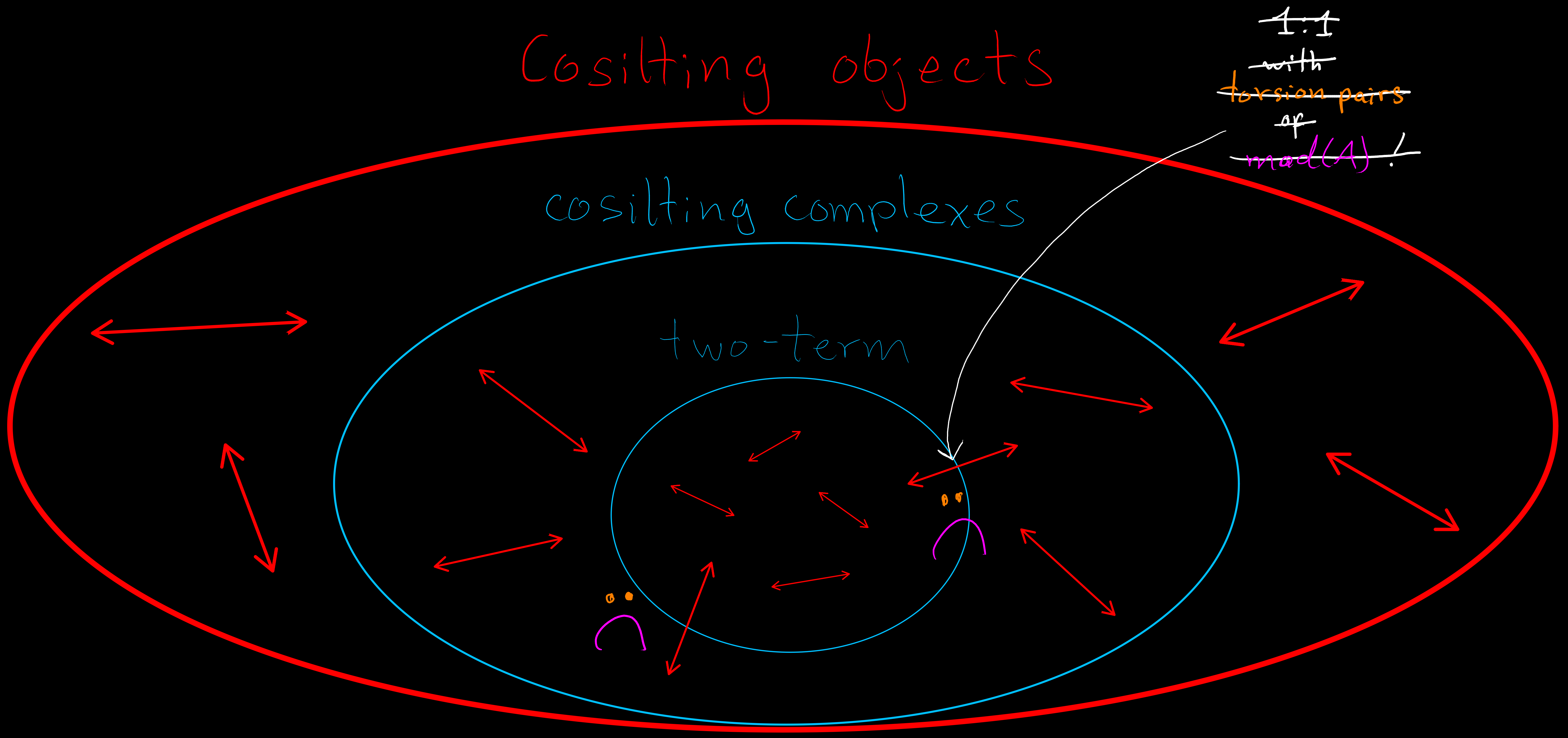
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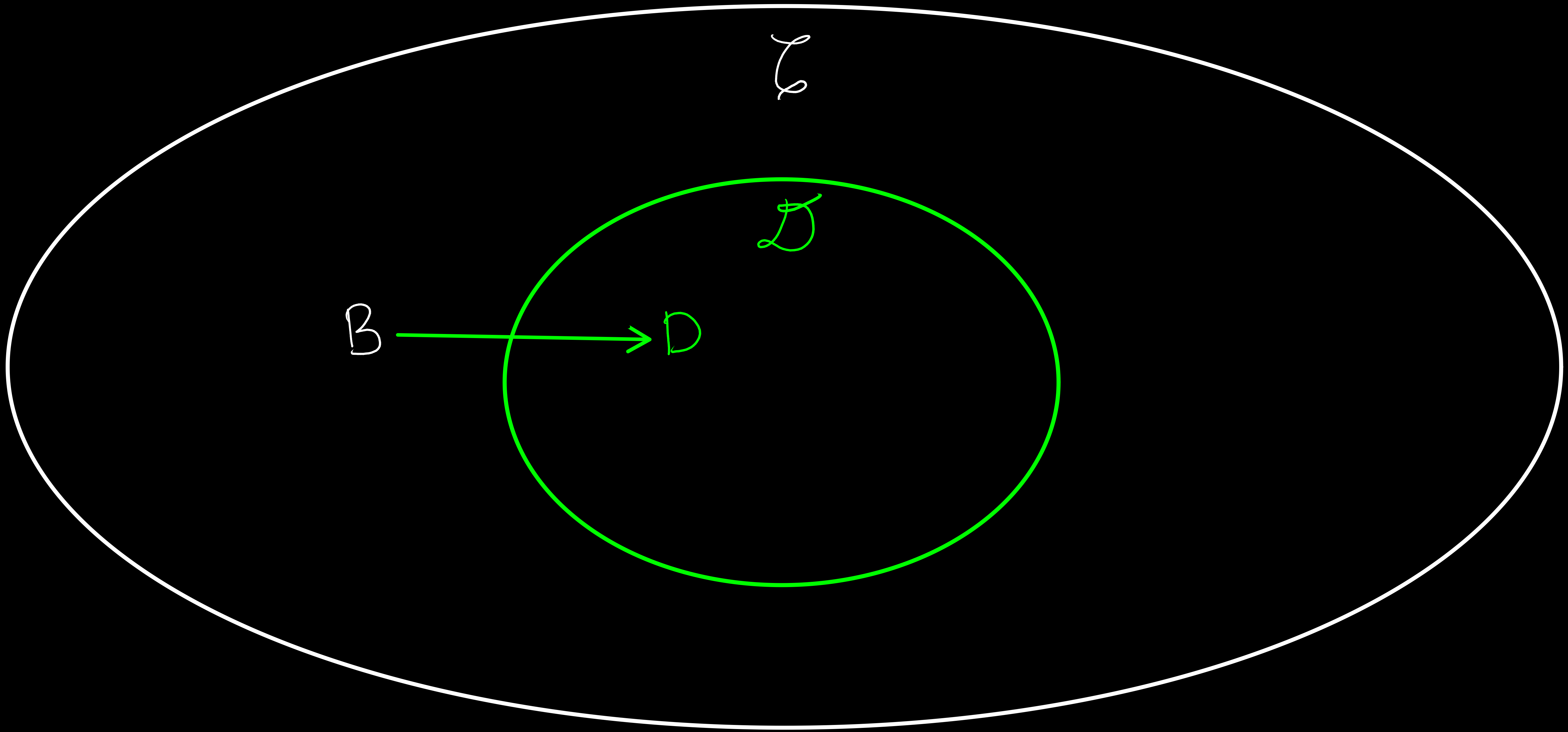

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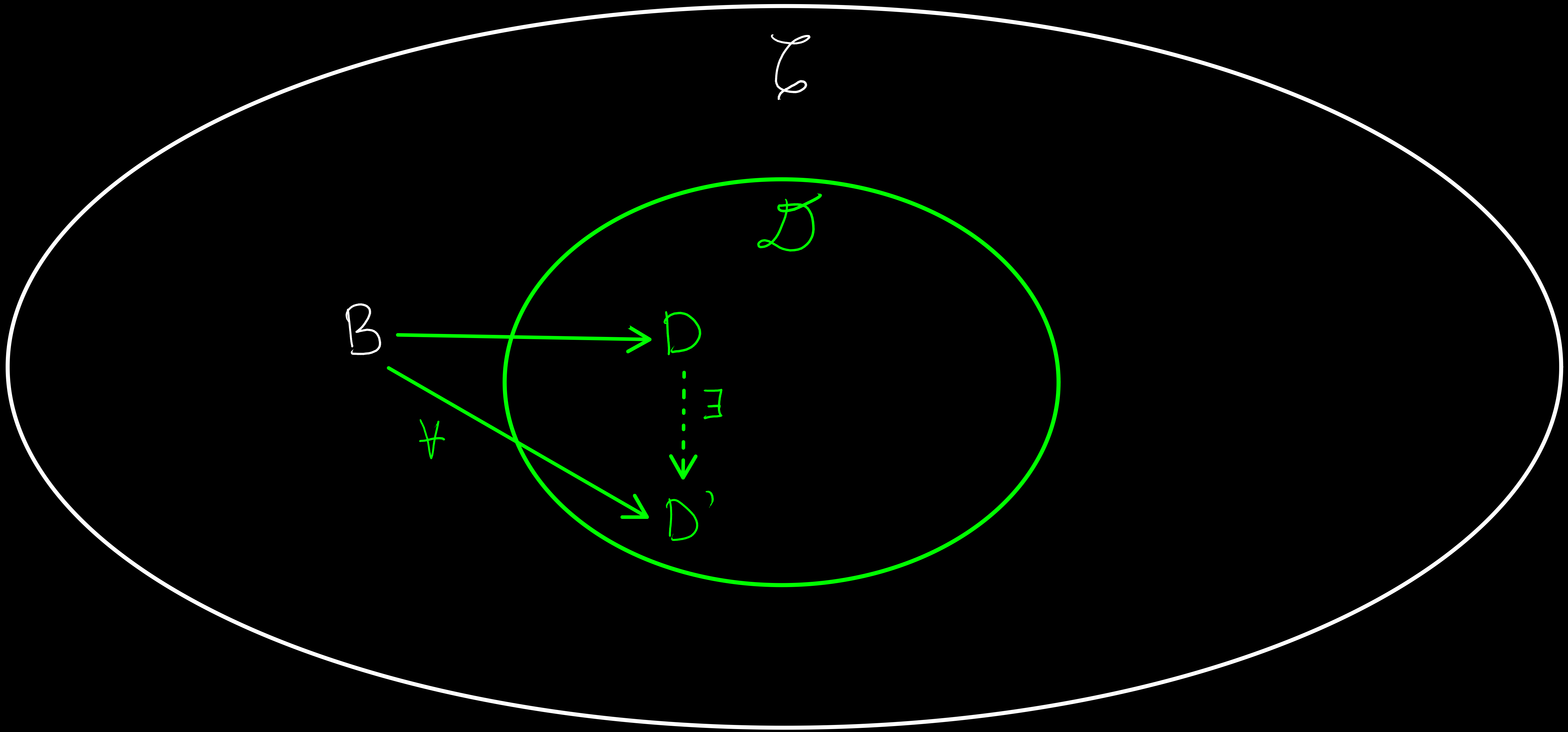

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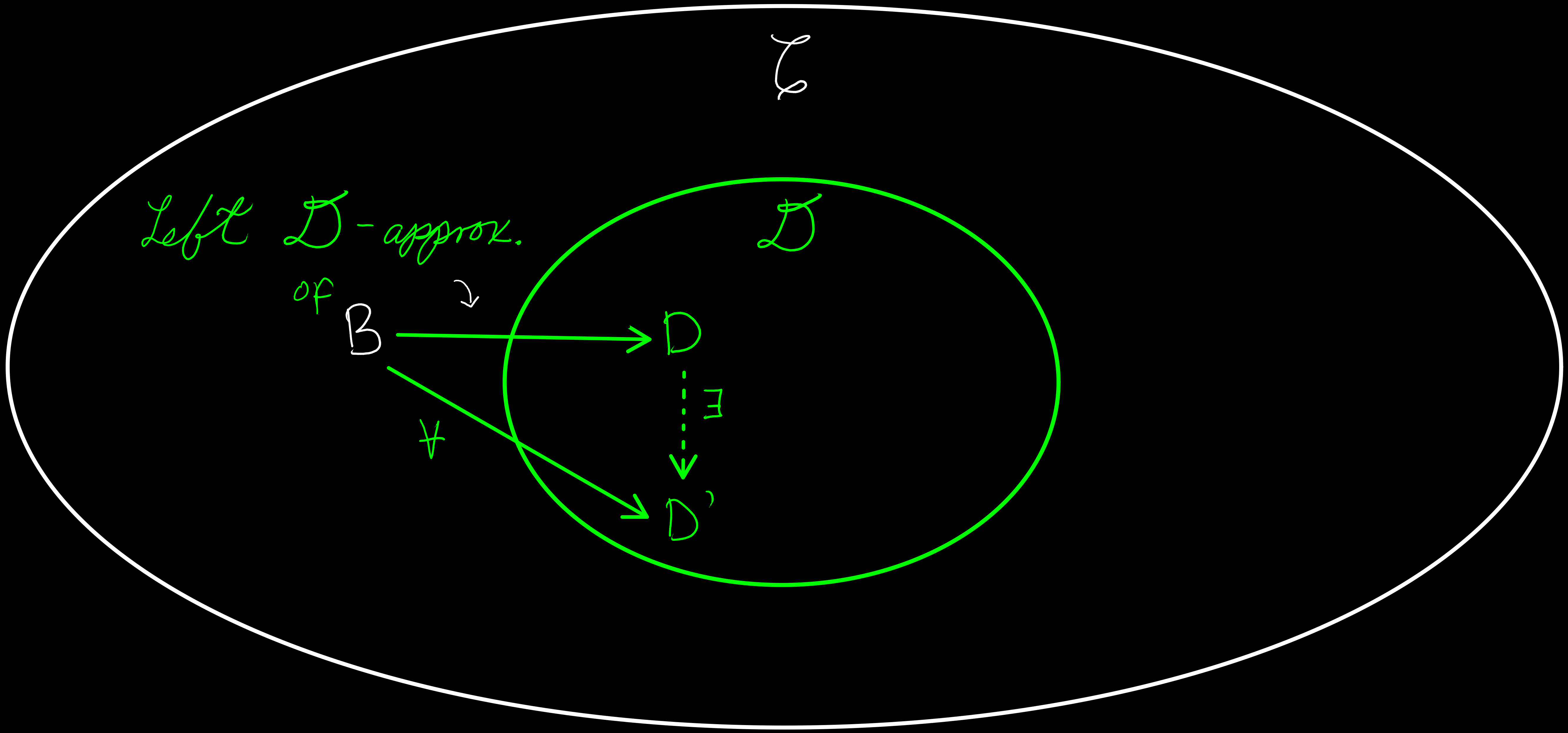

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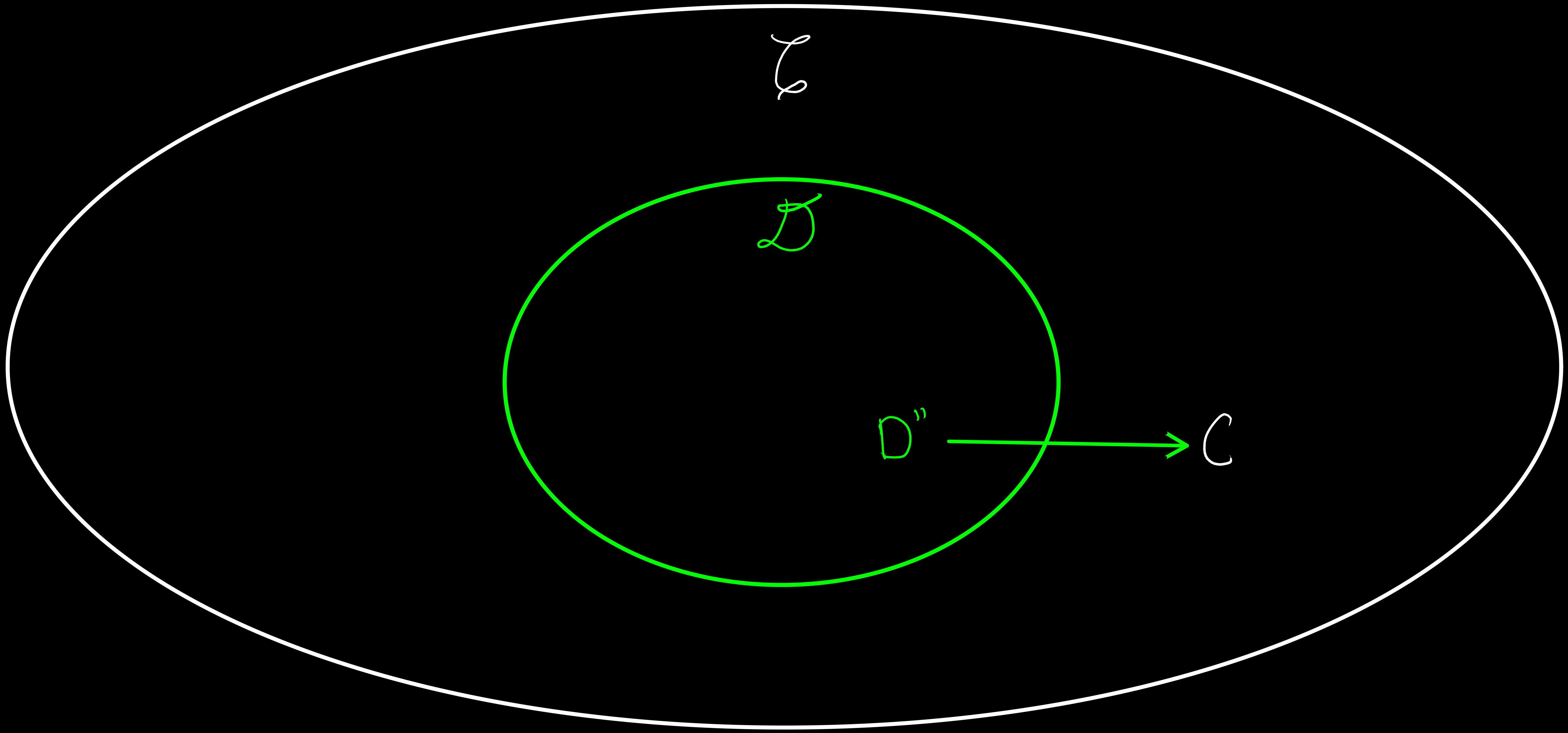

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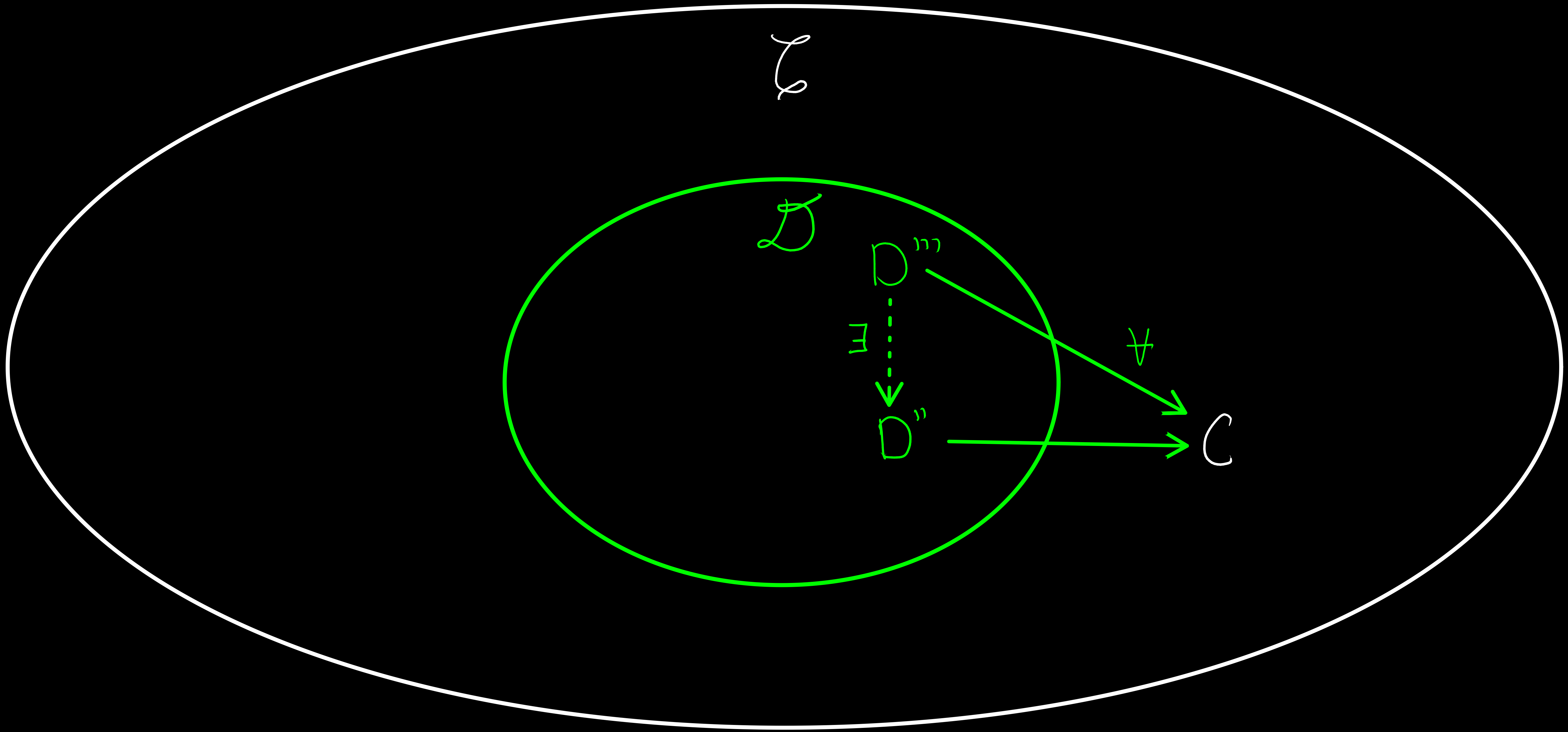

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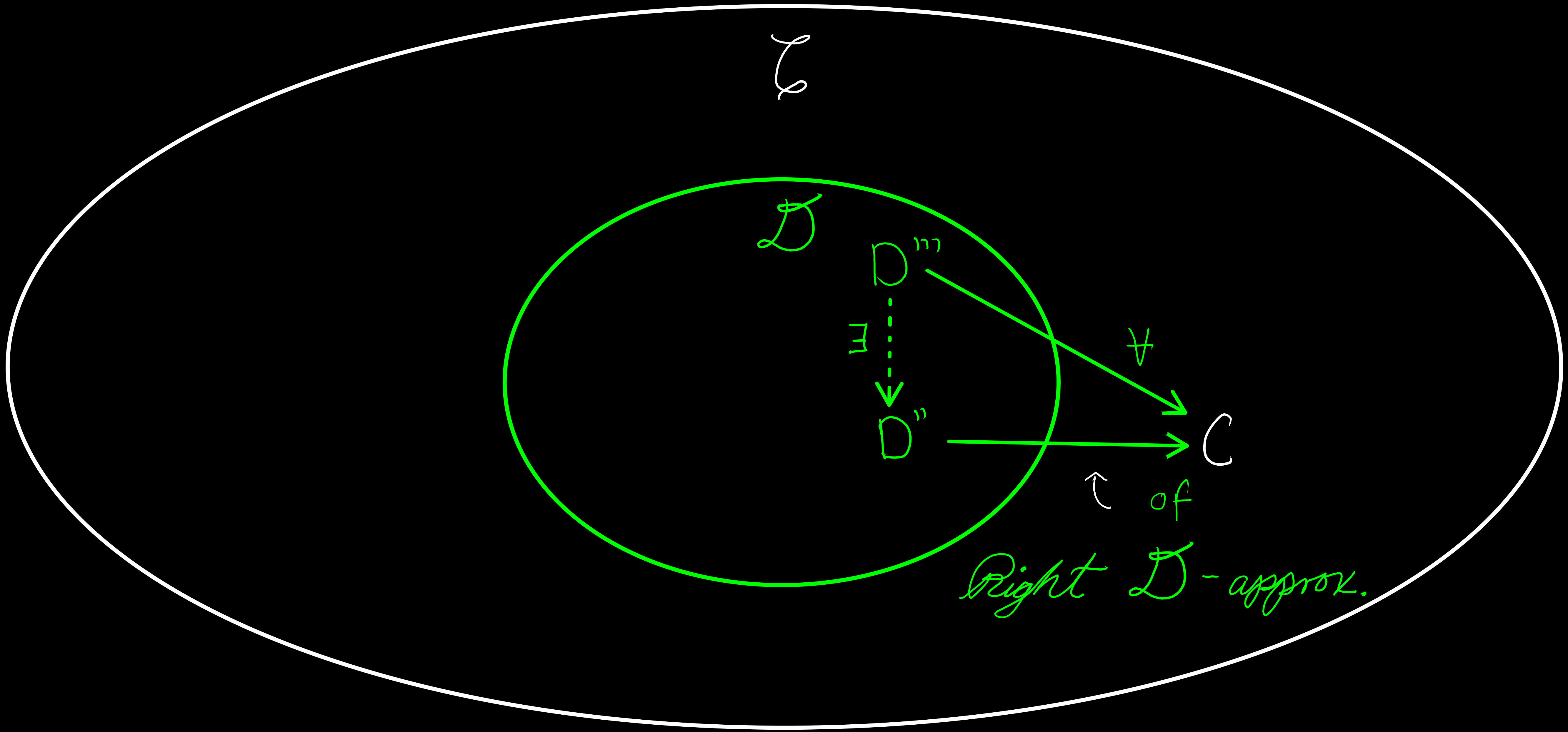

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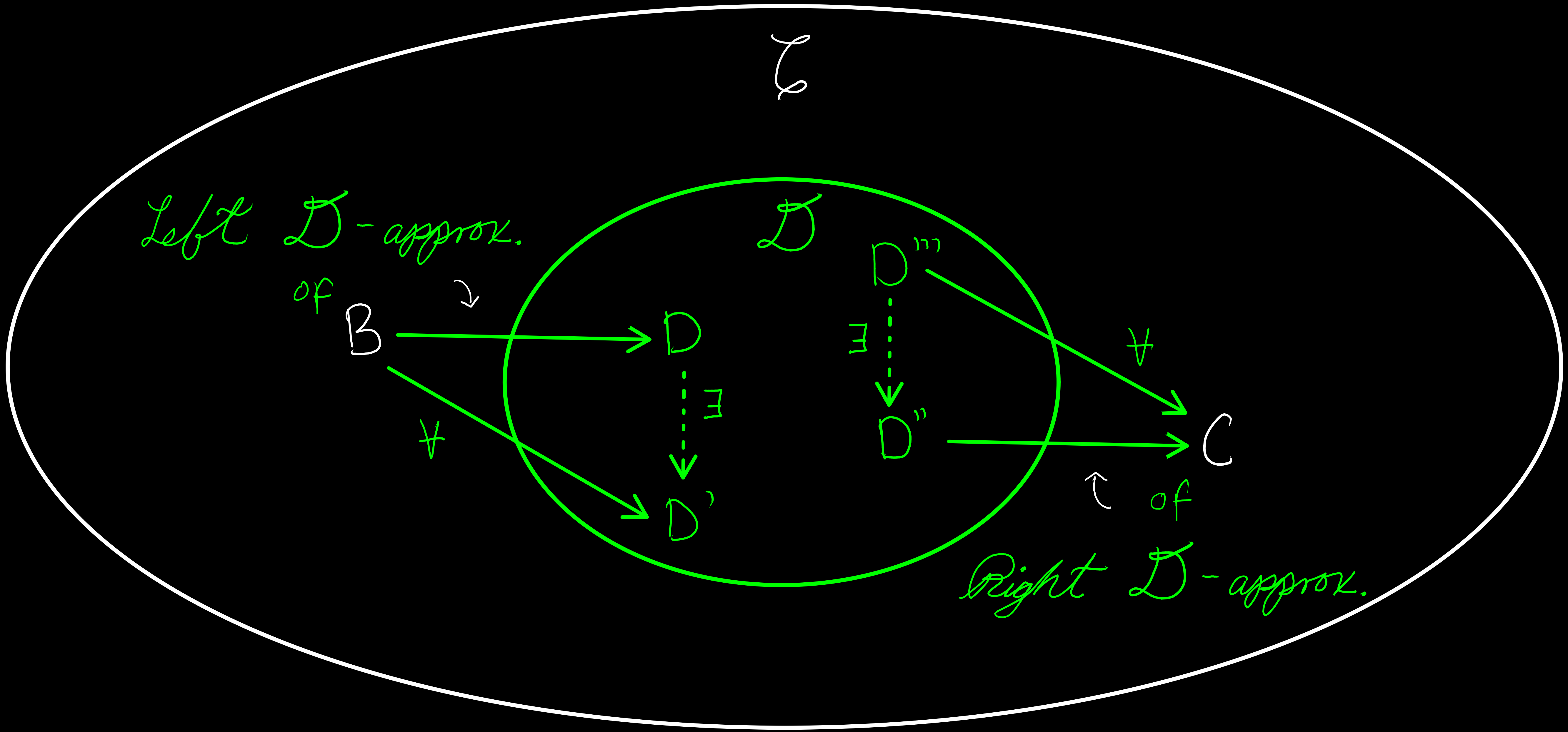

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$$\underbrace{\left(\bigoplus_{i=1, i\neq j}^{n} S_i\right) \oplus S_j'}_{\text{“left mutation of } S \text{ at } j\text{”}}, \text{ where } S_j \xrightarrow{\text{left add}(D)\text{-approx.}} \bigoplus_{i=1, i\neq j}^{n} S_i =: D \to S_j' \to S_j[1] \text{ in } K^b(\mathrm{proj}\, A).$$

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<u>Prp.</u> $S \in K^b(\mathrm{proj}\,A)$ is a silting cpx. $\Leftrightarrow$ $\mathrm{add}(S)$ is a silting subcat. of $K^b(\mathrm{proj}\,A)$.

Dfn. A complex $\omega \in D(\mathrm{Mod}(A))$ is *cosilting* if $\mathrm{Hom}_{D(\mathrm{Mod}(A))}(\omega^J, \omega[i]) = 0$ for any set $J$ and $i>0$ and the closure of $\omega$ under products, summands, extensions, cones and cocones $\mathrm{thick}(\mathrm{Prod}(\omega))$ is $K^b(\mathrm{Inj}\,A)$. Moreover, it is *two-term* if $\omega \in K^{[0,1]}(\mathrm{Inj}\,A)$.

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**Prp.** A complex $\omega \in D(\mathrm{Mod}(A))$ is *two-term cosilting* if, and only if, $\mathrm{Prod}(\omega)$ is a silting subcategory of $K^{[0,1]}(\mathrm{Inj}\,A)$.

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**Cor.** Let $\omega$ and $\omega'$ be *two-term cosilting* complexes and $\mathcal{D} = \mathrm{Prod}(\omega) \cap \mathrm{Prod}(\omega')$. TFAE.

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(a) $\omega'$ is a right (respectively, left) mutation of $\omega$.

(b) $\mathrm{Prod}(\omega')$ is a right (respectively, left) mutation of $\mathrm{Prod}(\omega)$ *with respect to* $\mathcal{D}$.

Děkuju!